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EFFECT OF SUCTION ON THIN FILM FLOW OF A THIRD GRADE FLUID IN A POROUS MEDIUM DOWN AN INCLINED PLANE WITH HEAT TRANSFER

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ABSTRACT - *In this paper, we have investigated the effect of suction on thin film flow of a third grade fluid in a porous medium down an inclined plane in presence of heat transfer. The method of regular and homotopy perturbations were used to obtain the solutions of the non-linear equations arising from the problem and the results presented graphically. The results revealed that both methods are in closed agreement as both almost have the same solution. Also, it was observed that as suction parameter increases, the temperature decreases, while increasing porosity parameter decreases the fluid velocity.*

Keywords: Third grade fluid, Brinkman number, Traditional perturbation method, Homotopy perturbation method, Suction Parameter, Inclined plane.

1 Introduction

The study of non-Newtonian boundary layer flow in a porous medium has in the past years become the main interest of researchers in various branches of sciences, engineering and technology, owing to its relevance and applications in industries. For instance applications of this so called differential type fluids are readily found from, wire coating, ink-jet print, the drilling of oil and gas wells to industrial processes involving waste fluids, synthetic fibres, food stuffs and the extrusion of molten plastics as well as in some flow of polymer solutions.

Most of these fluids either consist entirely of large molecules or have large molecules floating in them as well as particles or droplets. It is well known that material such as solutions and melts of polymer, plastics and synthetic fibres, certain oils and greases, soap and detergents, certain pharmaceutical and biological fluids fall into this category of non-Newtonian fluids.

However due to the complexity of these fluids, there are many models describing the properties but not all of non-Newtonian fluids, because the solution of flow problems of fluids in this class are more difficult to obtain. These models including the Navier-Stokes can not predict all the behaviour of non-Newtonian fluids, for example normal stress difference, shear thinning or shear thickening, shear relaxation, elastic and memory effects etc. In order to explain this behaviour of non-Newtonian fluids, Rivlin-Eriksen fluids of differential type are introduced. The third grade fluid model attempt to include such characteristics of viscoelastic fluids not exhibited by second grade fluids.

A coherent review of this study have been discussed by a list of reference including Sahoo and Do (2010), Elbashbeshy (2001), Pal (2010), Partha et al (2010), Makinde (2009), A.M. Siddiqui et al (2008), T. Hayat et al (2008), Aiyesimi et al (2012) and Khan N and Mahmood T. (2012).

Being inspired by these studies, the work Tahir M. and Nargis K. was extended with the aim of investigating the effect of suction parameter on a steady thin film flow of a third grade fluid in a porous medium down an inclined plane, with heat transfer analysis carried out. In most of this type of problems, the non-linear governing equations were solved using the readily available traditional perturbation method by several authors.

However, in this work, in addition to the use of the tradition perturbation method, the problems were equally solved using the method of the homotopy perturbation method, which appears to be simpler, straightforward, powerful and does not require the existence of any small or large parameter as it is the case of traditional perturbation method and the results compared.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The flow modeling is based on the modified Darcy's law, continuity equation and momentum equation for third grade fluid moving down an inclined plane given by:

$$\nabla P = -\frac{\mu V}{k_1} \tag{1}$$

$$\nabla \cdot V = 0 \tag{2}$$

$$\rho \frac{DV}{Dt} = -\nabla P + div\tau + \rho f - \frac{\mu V}{k_1} \tag{3}$$

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where;

ρ is the density, P is the pressure, μ is viscosity, k_1 is the Darcy permeability, V is velocity vector, ρf is the external body force, $\frac{D}{Dt}$ is material derivative, f is the external body force and τ is the Cauchy stress tensor which for a third grade fluid satisfies the constitutive equation

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 \left(A_1 A_2 + A_2 A_1\right) + \beta_3 (trA_1^2) A_1 \tag{5}$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1}\nabla V + (\nabla V)^{\perp} A_{n-1}, \quad n \geq 1 \tag{6}$$

where pI is the isotropic stress due to constraint incompressibility, μ is the dynamics viscosity, $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ are the material constants; $\perp$ indicate the matrix transpose, A_1, A_2, A_3 are the first three Rivlin-Ericken tensors and $A_0 = I$ is the identity tensor.

We consider a thin film of an incompressible MHD fluid of a third grade flowing in down an inclined plane. The ambient air is assumed stationary so that the flow is due to gravity and the porosity of the inclined plane.

By neglect the surface tension of the fluid and the film is of uniform thickness δ, we seek a velocity field of the form

$$V = [u(y), 0, 0,] \tag{7}$$

The flow is assumed steady with all the material parameters of fluids are assumed constant with $\alpha_i = 0$ *and* $\beta_i = 0$, then the equation (1) – (7) becomes

$$\frac{d^2u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} + \rho g \sin\theta - \frac{\mu u}{k_1} = 0 \tag{8}$$

Subject to boundary conditions

$$u = 0 \quad \text{at} \quad y = 0 \tag{10}$$

$$\frac{du}{dy} = 0 \quad \text{at} \quad y = \delta \tag{11}$$

Introducing the following dimensionless variables

$$\bar{u} = \frac{u\delta}{\mu}, \qquad \bar{y} = \frac{y}{\delta} \tag{12}$$

By using equation (12) in equation (8) - (11), we obtain

$$\frac{d^2u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} + K - \lambda u = 0 \tag{13}$$

The appropriate boundary condition can be constituted as follows:

$$u = 0 \quad \text{at} \quad y = 0 \tag{14}$$

$$\frac{du}{dy} = 0 \quad \text{at} \quad y = 1 \tag{15}$$

Equation (14) is the no slip condition and equation (15) comes from $\tau_{yx} = 0$ at $y = 1$.In the sequence, we take $\varepsilon = \beta$ and solve the system of equation (13)-(15) by the traditional perturbation method and also by the homotopy perturbation.

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation and work done due to deformation is given as

$$\rho C_p v \frac{dT}{dy} = k_2 \frac{d^2T}{dy^2} + \mu\left(\frac{du}{dy}\right)^2 + 2(\beta_2 + \beta_3)\left(\frac{du}{dy}\right)^4 \tag{16}$$

with boundary condition

$$T = T_w \quad \text{at} \quad y = 0 \tag{17}$$

$$T = T_\delta \quad \text{at} \quad y = \delta \tag{18}$$

where k_2 is the thermal conductivity, C_p represent the specific heat capacity, T is the fluid temperature, T_w is the temperature of he plane and T_δ is the temperature of the ambient fluid.

Introducing the following dimensionless variable

$$\bar{T} = \frac{T - T_w}{T_\delta - T_w} \tag{19}$$

The system of equation (16)-(19) after dropping the caps take the following form:

$$\frac{d^2T}{dy^2} - \Pr S \frac{dT}{dy} + Br\left(\frac{du}{dy}\right)^2 + 2Br\beta\left(\frac{du}{dy}\right)^4 = 0 \tag{20}$$

with thermal boundary condition

$$T = 0 \quad \text{at} \quad y = 0 \tag{21}$$

$$T = 1 \quad \text{at} \quad y = 1 \tag{22}$$

where

$\beta = \frac{\beta_2 + \beta_3}{\delta^4}\mu$ Viscoelastic parameter

$K = \frac{\rho g \sin\theta \delta^3}{\mu^2}$ Gravitational parameter

$\lambda = \frac{\delta^2}{k_1}$ Porosity parameter

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$B_r = \dfrac{\mu^3}{k_2\delta^2(T_2 - T_1)}$ is the Brinkman number

$\Pr = \dfrac{\mu C_p}{k_2}$ is the Prandtl number

$S = \dfrac{\rho\delta v}{\mu}$ is the Suction parameter

3. Solution of the problem

Solution by regular perturbation method

Let us assume ε as a small parameter in order to solve equation (13) – (15) by this method, we expand

$$u(y,\varepsilon) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) \tag{23}$$

Substituting equation (23) into equation (13) – (15) and rearranging based on powers of $\varepsilon -$terms.

We obtain the following problems of different order with their boundary conditions

Zeroth Order Problem

$$\frac{\partial^2 u_0}{\partial y^2} - \lambda u_0 + K = 0 \tag{24}$$

$$u_0 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 1 \tag{25}$$

First Order Problem

$$\frac{\partial^2 u_1}{\partial y^2} + 6\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_0}{\partial y^2} - \lambda u_1 = 0 \tag{26}$$

$$u_1 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 1 \tag{27}$$

Second Order Problem

$$\frac{\partial^2 u_2}{\partial y^2} + 12\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2} + 6\left(\frac{\partial u_0}{\partial y}\right)^2 \frac{\partial^2 u_1}{\partial y^2} - \lambda u_2 = 0 \tag{28}$$

with boundary condition

$$u_2 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial u_2}{\partial y} = 0 \quad \text{at} \quad y = 1 \tag{29}$$

Now we solve this sequence of problems and generate the following series solution

Zeroth order problem solution

$$u_0(y) = C_3 + C_4 y + C_5 y^2 + C_6 y^3 \tag{30}$$

First order problem solution

$$u_1(y) = C_{12} + C_{11}y + C_{10}y^2 + C_9 y^3 + C_8 y^4 + C_7 y^5 \tag{31}$$

Second – order problem solution

$$u_2(y) = C_{18} + C_{17}y + C_{16}y^2 + C_{15}y^3 + C_{14}y^4 + C_{13}y^5 \tag{32}$$

where C_i's are constant

Now, the solution series is given by

$$u(y) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y)$$

Next, we find the approximate solution for temperature distribution, for which we write

$$T(y,\varepsilon) = T_0(y) + \varepsilon T_1(y) + \varepsilon^2 T_2(y) \tag{33}$$

Substituting equation (33) into equation (20) –(22) and collecting the same power of ε, yields the following different order problems.

Zeroth Order Energy Problem

$$\frac{\partial^2 T_0}{\partial y^2} - \Pr S\frac{\partial T_0}{\partial y} + Br\left(\frac{\partial u_0}{\partial y}\right)^2 = 0 \tag{34}$$

$$T_0 = 0 \quad \text{at} \quad y = 0$$

$$T_0 = 1 \quad \text{at} \quad y = 1 \tag{35}$$

with the solution

$$T_0(y) = C_{19}y + C_{20}y^2 + C_{21}y^3 + C_{22}y^4 + C_{23}y^5$$
(36)

First Order Energy Problem

$$\frac{\partial^2 T_1}{\partial y^2} - \Pr S\frac{\partial T_1}{\partial y} + 2Br\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y} + 2Br\left(\frac{\partial u_0}{\partial y}\right)^4 = 0 \tag{37}$$

$$T_1 = 0 \quad \text{at} \quad y = 0$$

$$T_1 = 0 \quad \text{at} \quad y = 1 \tag{38}$$

with the solution

$$T_1(y) = C_{24}y^5 + C_{25}y^4 + C_{26}y^3 + C_{27}y^2 + C_{28}y \tag{39}$$

Second Order Energy Problem

$$\frac{\partial^2 T_2}{\partial y^2} - \Pr S\frac{\partial T_2}{\partial y} + 2Br\frac{\partial u_0}{\partial y}\frac{\partial u_2}{\partial y} + Br\left(\frac{\partial u_1}{\partial y}\right)^2 + 8Br\left(\frac{\partial u_0}{\partial y}\right)^3\frac{\partial u_1}{\partial y} = 0 \tag{40}$$

$$T_2 = 0 \quad \text{at} \quad y = 0$$

$$T_2 = 0 \quad \text{at} \quad y = 1 \tag{41}$$

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with the solution

$$T_2(y) = C_{33}y + C_{33}y^2 + C_{31}y^3 + C_{30}y^4 + C_{29}y^5 \quad (42)$$

Now, the solution series is given by

$$T(y) = T_0(y) + \varepsilon T_1(y) + \varepsilon^2 T_2(y)$$

Solution by Homotopy Perturbation Method

This section will be devoted to solve our initial problems arising from both momentum and energy analysis by homotopy perturbation technique with a view to compare our solutions with those earlier obtained by the regular perturbation method

The problem under consideration i.e equations (13)-(15) can be written as

$$L(v) - L(u_0) + qL(u_0) + q\left[\begin{array}{l} 6\beta\left(\frac{\partial v}{\partial y}\right)^2\left(\frac{\partial^2 v}{\partial y^2}\right) \\ + K - \lambda v \end{array}\right] = 0 \quad (43)$$

where $L = \frac{\partial^2}{\partial y^2}$ and $q \in [0,1]$ is the embedding parameters and u_0 is the initial guess approximation given as equation (30)

Also, Let

$$v(y) = v_0 + qv_1 + q^2 v_2 \quad (44)$$

Substitute equation (44) into (45) and equating the coefficient of like powers of q, we have the following

Zeroth Order Problem

$$\frac{\partial^2 v_0}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2} = 0 \quad (45)$$

$$v_0 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial v_0}{\partial y} = 0 \quad \text{at} \quad y = 1 \quad (46)$$

First Order Problem

$$\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2\left(\frac{\partial^2 u_0}{\partial y^2}\right) + K - \lambda v = 0 \quad (47)$$

$$v_1 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial v_1}{\partial y} = 0 \quad \text{at} \quad y = 1 \quad (48)$$

Second Order Problem

$$\frac{\partial^2 v_2}{\partial y^2} + 6\beta\left(\frac{\partial v_0}{\partial y}\right)^2 \frac{\partial^2 v_1}{\partial y^2} + 12\beta \frac{\partial v_0}{\partial y}\frac{\partial v_1}{\partial y}\frac{\partial^2 v_0}{\partial y^2} - \lambda v_1 = 0 \quad (49)$$

$$v_0 = 0 \quad \text{at} \quad y = 0$$

$$\frac{\partial v_1}{\partial y} = 0 \quad \text{at} \quad y = 1 \quad (50)$$

Now we solve this sequence of problems and generate the following series solution

Zeroth order problem solution

$$v_0(y) = C_6 y^3 + C_5 y^2 + (-3C_6 - 2C_5)y \quad (51)$$

First order problem solution

$$v_1(y) = C_{41}y^7 + C_{42}y^6 + C_{43}y^5 + C_{44}y^4 + C_{45}y^3 + C_{46}y^2 + C_{47}y \quad (52)$$

Second – order problem solution

$$v_2(y) = C_{48}y^{11} + C_{49}y^{10} + C_{50}y^9 + C_{51}y^8 + C_{52}y^7 + C_{53}y^6 + C_{54}y^5 + C_{55}y^4 + C_{56}y^3 + C_{57}y^2 + C_{58}y \quad (53)$$

The velocity field obtained by homotopy perturbation method is given as

$$u(y) = \lim_{q\to 1} v = \lim_{q\to 1}\left(v_0 + qv_1 + q^2 v_2\right). = v_0 + v_1 + v_2$$

Next, we find approximate solution of temperature profile using homotopy perturbation by written equation (20) as,

$$(1-q)\left[\frac{\partial^2 \theta}{\partial y^2} - \frac{\partial^2 T_0}{\partial y^2}\right] + q\left[\begin{array}{l} \frac{\partial^2 \theta}{\partial y^2} - \Pr S\frac{\partial \theta}{\partial y} + Br\left(\frac{\partial v}{\partial y}\right)^2 \\ + 2Br\beta\left(\frac{\partial v}{\partial y}\right)^4 \end{array}\right] = 0 \quad (54)$$

where $L = \frac{\partial^2}{\partial y^2}$ and $q \in [0,1]$ is the embedding parameters and T_0 is the initial guess approximately given as equation (36)

Also, Let

$$\theta(y) = \theta_0 + q\theta_1 + q^2\theta_2 \quad (55)$$

Substitute equation (55) into (54) and equating the coefficient of like powers of q, we have the following

Zeroth Order Energy Problem

$$\frac{\partial^2 \theta_0}{\partial y^2} - \frac{\partial^2 T_0}{\partial y^2} = 0 \quad (56)$$

$$\theta_0 = 0 \quad at \quad y = 0$$

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$$\theta_0 = 1 \qquad at \qquad y = 1 \tag{57}$$

with the solution

$$\theta_0 = C_{23} y^5 + C_{22} y^4 + C_{21} y^3 + C_{20} y^2 + \left(1 - C_{23} - C_{22} - C_{21} - C_{20}\right) y \tag{58}$$

First Order Energy Problem

$$\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 T_0}{\partial y^2} - \Pr S \frac{\partial \theta_0}{\partial y} + Br\left(\frac{\partial v_0}{\partial y}\right)^2 + 2Br\beta\left(\frac{\partial v_0}{\partial y}\right)^4 = 0 \tag{59}$$

$$\theta_1 = 0 \qquad at \qquad y = 0$$

$$\theta_1 = 0 \qquad at \qquad y = 1 \tag{60}$$

with the solution

$$\theta_1 = C_{68} y^{10} + C_{67} y^9 + C_{66} y^8 + C_{65} y^7 + C_{64} y^6 + C_{63} y^5 + C_{62} y^4 + C_{61} y^3 + C_{60} y^2 + C_{59} y \tag{61}$$

Second Order Energy Problem

$$\frac{\partial^2 \theta_2}{\partial y^2} + \Pr S \frac{\partial \theta_1}{\partial y} + 8Br\beta\left(\frac{\partial v_0}{\partial y}\right)^3 \frac{\partial v_1}{\partial y} + 2Br\frac{\partial v_0}{\partial y}\frac{\partial v_1}{\partial y} = 0 \tag{62}$$

$$\theta_2 = 0 \qquad at \qquad y = 0$$

$$\theta_2 = 0 \qquad at \qquad y = 1 \tag{63}$$

with the solution

with the solution

$$\theta_2(y) = C_{82} y^{14} + C_{81} y^{13} + C_{80} y^{12} + C_{79} y^{11} + C_{78} y^{10} + C_{77} y^9 + C_{76} y^8 + C_{75} y^7 + C_{74} y^6 + C_{73} y^5 + C_{72} y^4 + C_{71} y^3 + C_{70} y^2 + C_{69} \tag{64}$$

The temperature field obtained by homotopy perturbation method is given as

$$\theta(y) = \lim_{q\to 1}\theta = \lim_{q\to 1}\left(\theta_0 + q\theta_1 + q^2\theta_2\right).$$
$$= \theta_0 + \theta_1 + \theta_2$$

4. RESULTS AND DISCUSSION

A steady thin flow of a non-Newtonian third grade fluid through a porous medium over an inclined plane has been examined.

The governing non-linear ordinary differential equations involved are solved using the regular perturbation method as well as the homotopy perturbation method and both results were compared.

Figure 1 shows that results obtained from the methods is in good agreement for small value of (β). This shows that both methods have almost same solution, hence for further study, we then discuss the effect of various physical parameters on the velocity and temperature distribution, by placing their graphs side by side as illustrated from figure 2 through figure 6.

It is then found that the solution obtained by the regular perturbation and homotopy perturbation method are in fact the same for various value of suction parameter (S) and the Brinkman number (Br).

Figure 2 illustrate the effect of porosity parameter (λ) on the velocity distribution where it is observed that as λ increases, the velocity profile decreases. This of course is due to the fact that the porosity damping force also increases.

The fluid under study is made to flow down an inclined plane, so that the effect of the gravitational parameter is also investigated.

Figure 3 represents the consequent effect of parameter (K) on the velocity profile. Increasing the value of K resulted to increase in velocity. This is because, an increase in the value of K correspond to increase in the angle of inclination (θ) of the plane, which shows that as the angle of inclination θ increases, velocity is largely increase.

It is also observed from figure 4, the effect of suction parameter (S)on the temperature profile. It is noticed from the diagram, that the temperature profile decreases when (S), the suction parameter increases.

The temperature profile for various value of the Prandlt number $(\Pr)$ is shown in figure 5. The figure indicates that the temperature profile deceases with increase in Prandlt number. Thus the result is in agreement with physical interpretation that an increase in Prandlt number will cause the fluid to have a lower, thinner thermal boundary layer, thereby resulted to increase in temperature gradient.

Also figure 6 depict the effect of increasing the Brinkman number on temperature profile. The figure reveals that the temperature increases as the Brinkman number increases due to the heat generation through viscous and Joule dissipation.

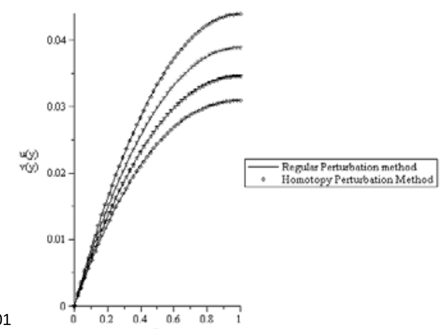

Figure 1: *Comparison of dimensionless velocity profile using Regular and Homotopy Perturbation for various values of* $\lambda = 0.2, 0.4, 0.6, 0.8$ $K = 0.1, \Pr = 5, S = 5, Br = 5, \beta = 1$

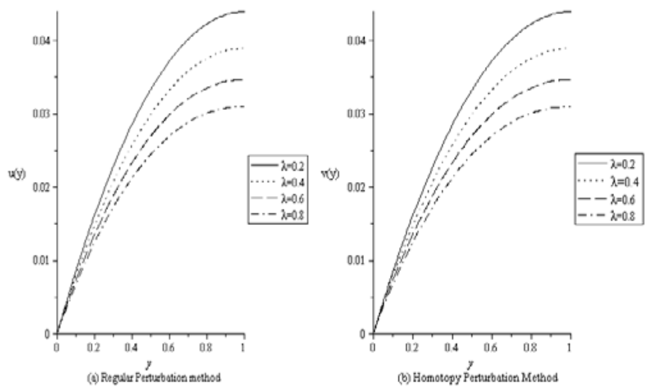

Figure 2: *Dimensionless velocity profile using Regular and Homotopy Perturbation for various values of* λ *when* $K = 0.1, \Pr = 5, S = 5, Br = 5, \beta = 1$

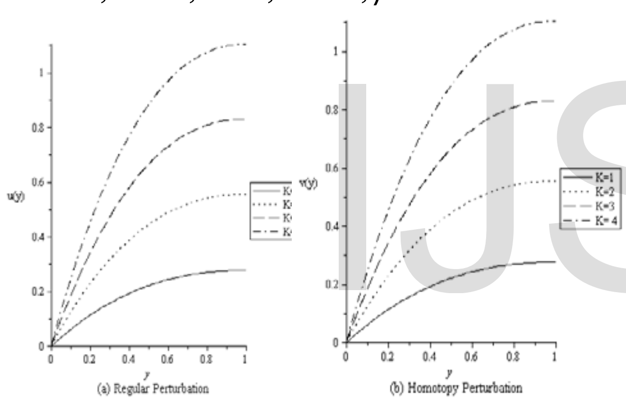

Figure 3: *Dimensionless velocity profile using Regular and Homotopy Perturbation for various values of* K *when* $S = 5, \Pr = 5, \lambda = 0.2, Br = 5, \beta = 1$

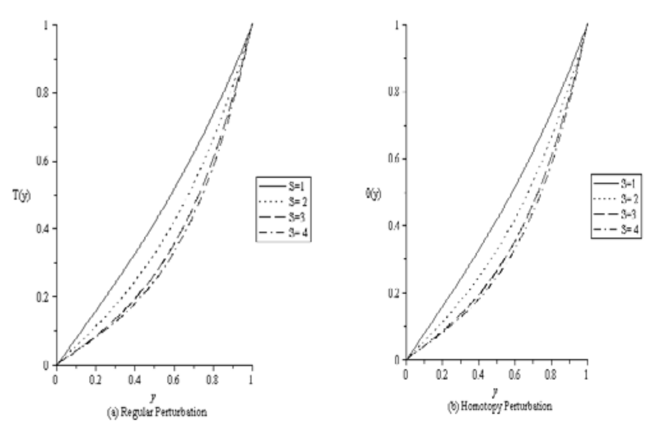

Figure 4: *Dimensionless Temperature profile using Regular and Homotopy Perturbation for various values of* S *when* $K = 0.1, \Pr = 5, \lambda = 0.2, Br = 5, \beta = 1$

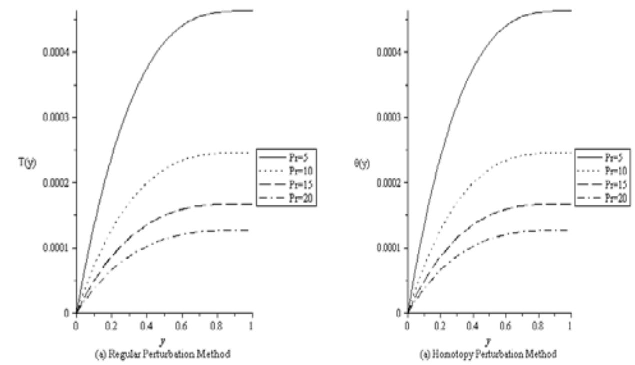

Figure 5: *Dimensionless Temperature profile using Regular and Homotopy Perturbation for various values of* $\Pr$ *when* $K = 0.1, S = 5, \lambda = 0.2, Br = 5, \beta = 1$

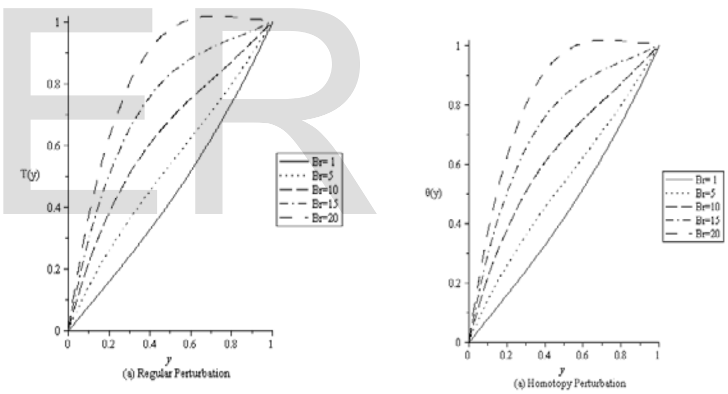

Figure 6: *Dimensionless Temperature profile using Regular and Homotopy Perturbation for various values of* Br *when* $K = 0.1,\ S = 5,\ \lambda = 0.2,\ \Pr = 5,\ \beta = 1$

5. CONCLUSION

The effect of suction parameter on a steady thin flow of a non-Newtonian third grade fluid over an inclined plane has been studied. The heat transfer analysis was carried out analytically, and the effect of porosity, suction and injection velocity on the velocity and temperature profiles was also investigated and the following conclusions were drawn

1. It is found that both the porosity and suction parameters have marked effect on both the velocity and temperature profiles
2. An increase in porosity parameter decreases the

velocity of the fluid

3. An increase in suction parameter resulted to decrease in temperature profile
4. A rise in the gravitational parameter consequently increases the inclination angle of the plane and so caused corresponding rise in fluid velocity
5. As the Prandlt number is increases, it generates a fall in the thermal boundary layer and heat escapes rapidly and thus lower fluid temperature
6. A rise in Brinkman number gives significant rise in the temperature of the fluid.

REFERENCES

Siddiqui A. M., Mahmood R. & Ghorn Q. K., (2008), "Homotopy Perturbation Method for thin film flow of a third grade fluid down an inclined plane", *Chaos solitons and fractals* **35**,140-147

Asghar S, Gulzar M.M, & Ayub M., (2006), "Effects of partial slip on flow of a third grade fluid", *Acta Mech Sinica* , **22**:393-396.

Sahoo B. and Do Y. (2010), Effect of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. Int. Comm. Heat Mass Trans. 10:1016.

Pal D. (2010), Mixed convection heat transfer in the boundary layers on an exponentially stretching surface with magnetic field. Appl. Math. Comp. 10:35

Elbashbeshy E. M. A, (2001), Heat transfer over an exponentially stretching continuous surface with suction. Arch. Mech. 53(6)643-651.

Miceel T.M. & Jamess M.H, (2008), "A note on the boundary layer equations with linear slip boundary condition", *Applied Mathematics letters* **21**, 810-813.

Ellehi R., (2009), "Effects of the slip boundary condition on non-Newtonian flows in a channel", *Commu Nonlinear Science Numerical Simul*, **14**,1377-1384.

Hayat. T, Saif S. & Abbas Z., (2008), "The influence of heart transfer in an MHD second grade fluid film over an unsteady stretching sheet", *Physics Letters* A372, 50037-5045.

Yongqi W. & Wu. W, (2007), "Unsteady flow of a fourth grade fluid to an oscillating plate", *International Journal of non-linear Mechanics* **42**, 432-441.

Makinde O. D. (2009), "Thermal critically for a reactive gravity driven thin film flow of a third grade fluid with adiabatic free surface down an inclined plane", *Appl. Math. Mech. Ed*. **30**(3), 373-380.

He, J. H. (2006) "Homotopy Perturbation Method for solving boundary value problem", *Phys. Letter A*, No.1-2, Vol. **350**.

He, J. H. (2003), "Homotopy Perturbation Method, a new nonlinear analytical techue", *Appl. Math. Comput* 135: 73-9.

He, J. H. (1999), "Homotopy Perturbation Technique", *Comput. Math. Appl. Mech. Eng*., **178**: 257-62.

Hsiao, K. (2011), "Unsteady mixed convection viscoelastic flow and heat transfer in a thin film flow over a porous stretching sheet with internal heat generation", *International Journal of the Physical Science*. Vol. **6**(22), pp. 5080-5090

Khan, N. & Mahmood, T. (2012), "The influence of slip condition on the thin film flow of a third order fluid", *International Journal of Non-linear Science*, Vol. **13**,No.1. pp. 105-116.

Chen, C. I., Chen C. K. & Yang Y.T. (2003), "Weakly non linear stability analysis of thin viscoelastic film flowing down on the outer surface of a rotating vertical cylinder", *International Journal of Engineering Science* **41**:1313-1336.

Aiyesimi, Y. M., Okedayo, G.T and Lawal O. W. (2013), " Unsteady MHD Thin film flow of a third grade fluid with heat transfer and no slip boundary Down an inclined plane", International Journal of Scientific and Engineering Research., Vol. 4, Issue 6, pp420-428.